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GEOMETRY.

SOME PROPOSITIONS ON THE REGULAR DODECAHEDRON.

By **ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Miss.**

(1). The edges are parallel, in pairs.

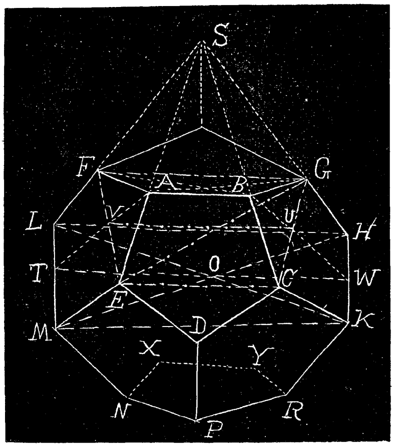
To prove *HK* parallel to *LM*.

Since the faces are equal regular pentagons *FG* and *EC* are equal.

They are also parallel, each being parallel to *AB*.

Hence *GC* and *FE* are equal and parallel.

Therefore *HK*, which is parallel to *GC*, and *LM*, which is parallel to *FE*, are parallel.



(2). The planes of the faces are parallel, in pairs.

To prove the plane of face *AG* parallel to the plane of face *NR*.

As in (1) *GC* was proved parallel to *FE*.

So may *FB* be proved parallel to *MD*, and, therefore, parallel to *NP*.

Similarly, *AG* and *PR* are parallel.

Therefore, the planes of faces *NR* and *AG* are parallel.

(3). Edges in different faces extending from vertices of the same face are inclined to each other at angles of 60°.

To prove *HG* and *LF* inclined to each other at an angle of 60°.

EA and *CB* produced intersect at *S*.

From the similar triangles *SAB* and *SEC*, $SA:AB=SE:EC$.

Representing an edge of the dodecahedron by a, and a diagonal of a face by d, this proportion gives $SA=\frac{a^2}{d-a}$.

But $d=\frac{a}{2}[\sqrt{5}+1]$. Therefore, $SA=\frac{a}{2}[\sqrt{5}+1]=d$.

That is, *CB* produced meets *EA* produced at a point distant d from *A*.

Similarly *LF* produced meets *EA* produced at a point distant d from *A*; hence, at *S*.

In this way it can be shown that *HG* produced passes through *S*.

Hence *LF* and *HG* are in the same plane; and triangle *SFG* is equilateral, each side being equal to d.

Therefore *LF* and *HG* are inclined to each other at an angle of 60°.

(4). If the ten diagonals of two non-adjacent, non-parallel, faces being grouped in pairs, both faces being represented in every pair, (a) the diagonals of one pair are parallel, (b) those of each of two pairs meet, when produced, at angles of 60°, and (*c*) those of each of two other pairs are inclined to each other at angles of 36°.

Consider the faces *BK* and *AM*.

(*a*). *CG* is parallel to *EF* (proved in the demonstration of (1)).

(*b*). *KB* and *MA* lie in the same plane.

For *AB* is parallel to *FG*; and *FG* is parallel to *LH*, since *FG* divides the sides of the triangle *SLH* proportionally; and *LH* is parallel to *MK*, since *LM* and *HK* and equal and parallel.

Hence *AB* and *MK* are parallel and determine a plane.

KB and *MA* are parallel to *HG* and *LF*, respectively.

Therefore, when, produced, they meet at an angle of 60°.

Similarly with *HB* and *LA*.

(*c*). *KG* and *MF* are in the same plane, since *FG* and *MK* are parallel.

And they are parallel to *CB* and *EA*, respectively.

The angle between them, therefore, equals angle *ASB*, or 36°.

So with *HC* and *LE*.

(5). To express the diagonals in terms of a and d.

One of the shortest is *EG*, one of the longest is *LK*, and a medium diagonal is *LH*.

Denote these by s, l, and m, respectively.

LH is a side of the equilateral triangle *SLH*.

Hence, $m = SF + FL = \frac{a}{2}[\sqrt{5}+1] + a$

$$= \frac{a}{2}[\sqrt{5}+3] = \frac{1}{a}\left[\frac{a}{2}[\sqrt{5}+1]\right]^2 = \frac{d^2}{a}.$$

To determine s and l it is necessary to show that *LMKH* is a rectangle, and *FECG* a square.

The plane bisecting *LM* at right angles contains all points equidistant from *L* and *M*, and is perpendicular to all lines parallel to *LM*.

Therefore this plane will pass through the middle point of *FE*, through points *A* and *B*, and, hence, the line *AB*, through the middle points of *GC* and *HK*, intersecting the planes *LK* and *FC* in the lines *TW* and *UV*, respectively.

Since *AB* is parallel to *FG* and *LH*, it is parallel to planes *FC* and *LK*, and, hence, to *UV* and *TW*.

FG, *UV*, *LH*, and *TW*, then, are parallel, and since *FE* and *LM* are respectively perpendicular to *UV* and *TW*, they are perpendicular to *FG* and *LH* respectively.

It follows that *FC* is a square and *LK* a rectangle.

From right triangle *ECG*, $s = \frac{a\sqrt{2}}{2}[\sqrt{5}+1] = \sqrt{2}\,d.$

From right triangle *LMR*, $l = \frac{a\sqrt{3}}{2}[\sqrt{5}+1] = \sqrt{3}\,d.$

This result may be written $\sqrt{\frac{a}{2}[3+\sqrt{5}]3a} = \sqrt{3am}.$

It may be remarked that d is a mean proportional between a and m, l is a mean proportional between $3a$ and m, and l is the diagonal of a cube whose face-diagonal is s and edge d.

(6). To determine a point equidistant from all the vertices.

LK bisects MH at O.

In the rectangle of which GH and MN are opposite sides, GN bisects MH, therefore passes through O and is itself bisected at O.

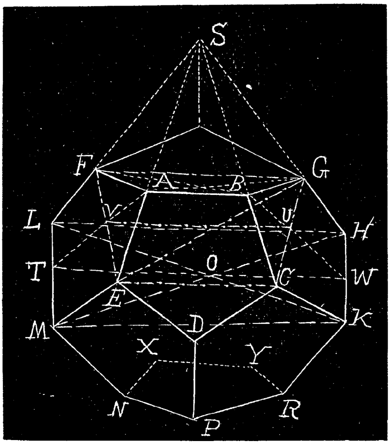
So with all the longest diagonals.

Hence O is equidistant from all the vertices.

COROLLARY 1. O is equidistant from all the edges.

COROLLARY 2. O is equidistant from all the faces.

(7). To compute the volume.



This could be done by conceiving the dodecahedron composed of twelve equal pyramids with O as their common vertex and the faces for bases. But another method will be adopted.

The dodecahedron is composed of a cube one of whose faces is $EFGC$ and six equal truncated triangular prisms, one of which has AB, FG, and EC for its lateral edges. A right section of this truncated prism is a triangle with base equal to EF, and altitude, the perpendicular from A to UV. Since $ABXY$ is a rectangle equal to $LMKH$, this perpendicular is one-half the difference between $AX(=LH)$ and an edge of the cube, and, therefore, equals $\frac{1}{2}a$.

Hence the area of the right section is $\frac{a^2}{8}[\sqrt{5}+1]$, and the volume of the truncated prism is $\frac{a^2}{8}[\sqrt{5}+1]\frac{a+a[\sqrt{5}+1]}{3}$.

The sum of the volumes of six such solids is $\frac{a^3}{4}[7+3\sqrt{5}]$.

Volume of cube$=\frac{a^3}{4}[8+4\sqrt{5}]$.

Volume of dodecahedron$=\frac{a^3}{4}[15+7\sqrt{5}]$.

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134. Proposed by J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.

If $ABCD$ is a quadrilateral circumscribing a circle, show that the line joining the middle points of the diagonals AB, CD passes through the center of the circle.

Solution by HARRY S. VANDIVER, Bala, Penna.

We will use quadrilinear coördinates denoting the equation of the four